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OPTIMAL SOLUTION TO FULLY FUZZY TIME COST TRADE OFF PROBLEM

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ABSTRACT

Time Cost Trade Off problem is one of the main aspects of project scheduling. The Method of solving these kinds of problems requires a scheduling with more stability against environmental variations. In this paper, we propose a new solution procedure for time cost trade off problem in which both times and costs are fuzzy. By using a modified subtraction we propose a method for finding an optimal duration by crashing the fuzzy activities of a project network without converting the fuzzy activity times and costs to classical numbers. Finally, illustrative examples are provided to demonstrate the efficiency of the proposed method.

KEYWORDS: Project Scheduling, Time Cost Trade Off, Triangular Fuzzy Number

AMS Subject Classification: 94D05, 03B52, 03E72, 28E10

1. INTRODUCTION

An important aspect of project management is scheduling time accurately. This is critical component of project planning as this will the deadline for the completion of a project. Since the late 1950's critical Path Method techniques have become widely recognized as valuable tools for the planning and scheduling of projects. But in many cases, project should implement before the data that was calculated by Critical Path Method. Achieving this goal, can be used more productive equipment or hiring more workers.

Reducing the original project duration which is called crashing PERT/CPM networks in many studies which is aimed at meeting a desired deadline with the lowest amount of cost is one of the most important and useful concepts for project managers. Since there is a need, to allocate extra resources in PERT/CPM crashing networks and the project managers are intended to spend the lowest possible amount of money and achieve the maximum crashing time, as a result both direct and indirect costs will be influenced in the project; therefore in some researches the term 'time-cost trade-off 'is also used for this purpose.

Several approaches are proposed over the past years for finding the optimum duration with minimum cost. In many researches, programming models are developed to solve optimally the trade off among time, cost and quality. For examples Cusack, 1985 and Babu and Suresh 1996 and Demeulemeester et al. 1996 were used linear programming and dynamic programming models are presented to crash projects.

Some authors have claimed that fuzzy set theory is more appropriate to model these problems. Wang et al. 1993 developed a model to project scheduling with fuzzy information. Leu et al. 1999 developed a fuzzy optimal model to formulate effects of both certain activity duration and resource constraint. Arican and Gungorpresented fuzzy goal programming model for time-cost trade off problem. Leu et al. 2001 proposed a new fuzzy optimal time cost trade off method and GA based approach to solve it. Guang et al. 2005 presented a new solution approach for fuzzy time-cost trade off model based on Genetic Algorithm. Ghazanfari et al. 2007 developed a new possibilistic model to determine optimal duration for each activity in the form of triangular fuzzy number. Also Yousefli et al. 2008 presented a heuristic method to solve a project scheduling problem by using decision making in fuzzy environment. ShakeelaSathish, 2012 proposed a new approach to solve fuzzy network crashing problems.

In this paper, we have presented a new solution procedure for time-cost trade off problem in fuzzy environment. We have considered time cost trade off problem in uncertain environment in which normal and crash durations of each activity are considered uncertain and shown in the form of trapezoidal fuzzy numbers. Optimum durations of activities are calculated in the form of trapezoidal fuzzy number. Finally to test the applicability of the method, suitable numerical examples have been dealt with.

2. PRELIMINARIES

In this section, some basic definitions of fuzzy theory have been defined by Kaufmann and Gupta and Zimmermann, are presented.

Definition 2.1

A fuzzy set $\widetilde{A}$ is a set of ordered pairs, $\{(x, \mu_{\widetilde{A}}(x) / x \in R\}$ where $\mu_{\widetilde{A}}(x) : R \rightarrow [0,1]$ and is upper semi-continuous. Function $\mu_{\widetilde{A}}(x)$ is called membership function of the fuzzy set.

Definition 2.2

A fuzzy set $\widetilde{A}$ is called positive if its membership function is such that $\mu_{\widetilde{A}}(x) = 0 \ \forall \ x \leq 0.$

Definition 2.3

A fuzzy set $\widetilde{A}$ defined on the set of real numbers R is said to be a fuzzy number if its membership function has the following characteristics:

- $\mu_{\widetilde{A}}(x) : R \rightarrow [0,1]$ is continuous.
- $\mu_{\widetilde{A}}(x) = 0$ for all $(-\infty, a] \cup [c, \infty).$
- $\mu_{\widetilde{A}}(x)$ is strictly increasing on [a,b] and strictly decreasing on [b,c].
- $\mu_{\widetilde{A}}(x) = 1$ for all $x \in b$ where $a \leq b \leq c.$

Definition 2.4

A fuzzy number $\widetilde{A} = (a, b, c)$ is said to be a triangular fuzzy number if its membership function is given by,

$$\mu_{\widetilde{A}}(x) = \begin{cases} \dfrac{(x-a)}{(b-a)} & , a \leq x \leq b \\ \dfrac{(x-c)}{(b-c)} & , b \leq x \leq c \end{cases}$$

We use F(R) to denote the set of all triangular fuzzy numbers.

Definition 2.5

A fuzzy number $\widetilde{A} = (a, b, c, d)$ is said to be a trapezoidal fuzzy number if its membership function is given by,

$$\mu_{\tilde{A}}(x)=\begin{cases}\dfrac{x-a}{b-a} & , a\le x<b\\ 1 & , b\le x\le c\\ \dfrac{d-x}{d-c} & , c<x\le d\\ 0 & , x<a \quad and \quad d<x\end{cases}$$

2.1 Arithmetic Operations on Trapezoidal Fuzzy numbers

Arithmetic operations between two triangular fuzzy numbers $\tilde{A}=(a_1,b_1,c_1,d_1)$ and $\tilde{B}=(a_2,b_2,c_2,d_2)$ are:

- $\tilde{A}+\tilde{B}=(a_1+a_2,b_1+b_2,c_1+c_2,d_1+d_2)$
- $\tilde{A}-\tilde{B}=(a_1-d_2,b_1-c_2,c_1-b_2,d_1-a_2)$
- $\tilde{A}.\tilde{B}=(a_1a_2,b_1b_2,c_1c_2,d_1d_2)$
- $\tilde{A}/\tilde{B}=(a_1/d_2,b_1/c_2,c_1/b_2,d_1/a_2)$

Definition 2.1.1: Modified Subtraction

By reviewing the approach followed by Ahmad Soltani and Rasoul Haji 2007 [1], it is felt that the definition adopted to their fuzzy problem will also suit well for the problem under this study.

If $\tilde{A}=(a_1,b_1,c_1,d_1)$ *and* $\tilde{B}=(a_2,b_2,c_2,d_2)$,

then $\tilde{C}=\tilde{A}-\tilde{B}$ *where* $\tilde{C}=(c^1,c^2,c^3,c^4)$

$$\begin{aligned}c^4 &= \max(0,(d_1-d_2))\\ c^3 &= \max(0,\min(c^4,(c_1-c_2)))\\ c^2 &= \max(0,\min(c^3,(b_1-b_2)))\\ c^1 &= \max(0,\min(c^2,(a_1-a_2)))\end{aligned}$$

2.2 Ranking of Fuzzy Number

Let F(R) denotes the set of all triangular fuzzy numbers. Let us define a ranking function $\Re : F(R)\to R$ which maps all triangular fuzzy numbers into R.

If $\tilde{A}=(a,b,c,d)$ is a triangular fuzzy number, then the Graded Mean Integration Representation (GMIR) method to defuzzify the number is given by,

$$\Re(\tilde{A})=\frac{a+2b+2c+d}{6}$$

2.3 Fuzzy Project Network

A fuzzy project network is an acyclic digraph, where the vertices represent events and the directed edges represents activities, to be performed in a project. We denote this fuzzy project network by $\widetilde{N} = < V, A, \widetilde{D} >$. Let $V = \{v_1, v_2, \ldots v_n\}$ be the set of all vertices (events), where v_1 *and* v_n are the tail and head events of the project. Let $A \subset V \times V$ be the set of all directed edges, $A = \{a_{ij} = (v_i, v_j) / v_i, v_j \in V\}$, that represents the activities to be performed in the project.

A Critical Path is a longest path from the initial event v_1 to the terminal event v_n of the project, and an activity a_{ij} on a critical path is called a critical activity.

3. THE PROCEDURE FOR PROPOSED METHOD

The procedure to find the optimum solution to the given problem is as follows:

Step 1: Determine the critical path of the given project.

Step 2: Find the total normal duration and project cost using the formula

Project cost= (Direct Cost+ (Indirect cost*project duration))

Step 3: Find the minimum cost slope by the formula:

Cost Slope = (Normal cost-Crash cost) / (Normal time-Crash time)

It is to be observed that the subtraction in the above expression refers to the modified subtraction as per definition 2.1.1.

Step 4: Determine the crash time and crash cost for each activity to compute the cost slope.

Step 5: Identify the activity with the minimum cost slope and crash that activity. Identify the new Critical path and find the cost of the project by formula,

Project cost = (Project Direct cost + Crashing cost of crashed activity) + Indirect cost*Project duration)

Step 6: Crash all activities in the project simultaneously.

Step 7: After crashing all activities, determine the Critical Path and non Critical Paths, also identify the critical activities.

Step 8: In the new Critical path select the activity with the next minimum cost slope, and repeat this step until all the activities along the critical path are crashed up to desired time.

Step 9: At this point all the activities are crashed and further crashing is not possible. The crashing of non critical activities does not alter the project duration time and is of no use.

NUMERICAL ILLUSTRATIONS

Example

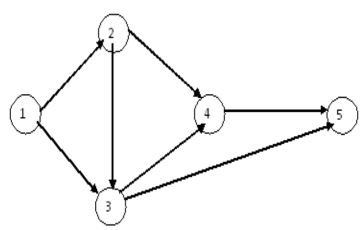
We have considered the project on launching of a new product is presented with a 7 activity CPM network illustrating project scheduling with time-cost trade-off problem. The precedence relationship of the network is given in Figure 1.The durations direct and indirect cost rates for both normal and crash modes of each activity are shown in Table 1.Only integer activity times have been considered to estimate project duration.

Table 1: Details of the Project

Activity No.	Activity	Name of the Activity	Crash Time	Normal Time
A	$1 \to 2$	Forecast sales volume, using market research	(15,17,20,24)	(25,27,30,34)
B	$1 \to 3$	Design pre-paid meter	(10,12,15,18)	(13,16,20,25)
C	$2 \to 3$	Materials and component procurement	(15,16,20,24)	(16,20,26,30)
	$2 \to 4$	Laboratory test of materials and component	(2,4,5,8)	(5,7,8,12)
E	$3 \to 4$	Proto-type production	(3,5,6,8)	(4,6,7,10)
F	$3 \to 5$	Laboratory and commercial testing	(2,3,4,9)	(3,5,7,11)
G	$4 \to 5$	Commercial production sales and distribution planning	(3,5,6,10)	(4,6,9,12)

Table 2: Details of the Project

Activity No.	Activity	Crash Cost	Normal Cost
A	$1 \to 2$	(300,400,450,550)	(150,200,250,300)
B	$1 \to 3$	(200,250,280,340)	(100,120,140,180)
C	$2 \to 3$	(200,250,300,350)	(80,120,160,200)
D	$2 \to 4$	(150,220,280,350)	(50,90,120,250)
E	$3 \to 4$	(350,450,520,600)	(100,160,210,260)
F	$3 \to 5$	(140,225,350,460)	(50,90,130,180)
G	$4 \to 5$	(150,210,250,320)	(50,100,140,190)



The critical path of the above problem is: $1 \to 2 \to 3 \to 4 \to 5$

The direct cost of the given project is: (570, 875, 1150, 1560) and total duration is (49, 59, 72, 86).

The total cost of the project is: (5470, 6775, 8350, 10160).

The calculation of the slope cost for each activity is given in the following table:

Table 3: Calculation of Slope Cost

Activities	ΔT	ΔC	Slope Cost($\Delta C / \Delta T$)
$1 \to 2$	(10,10,10,10)	(150,200,200,250)	(15,20,20,25)
$1 \to 3$	(3,4,5,7)	(100,130,140,160)	(14.3,26,35,53.3)
$2 \to 3$	(1,4,6,6)	(120,130,140,150)	(20,21.7,35,150)
$2 \to 4$	(1,1,1,2)	(100,100,100,100)	(25,33.3,33.3,33.3)
$3 \to 4$	(2,2,2,3)	(250,290,310,340)	(60,90,100,100)
$3 \to 5$	(1,2,2,2)	(90,135,220,280)	(45,67.5,110,280)
$4 \to 5$	(1,1,2,2)	(100,110,110,130)	(50,55,110,130)

From the above calculation, the minimum slope cost is occurring in two activities. We start to crash the activities from the minimum one. The steps are given below:

Table 4: Calculation of Minimum Duration and Cost

Stage	Crash	Total Duration	Total Cost
1	1 → 2by (10,10,10,10)	(39,49,62,76)	(4620,5975,7550,9410)
2	2 → 3 by (1,4,6,6)	(38,45,56,70)	(4490,5505,6890,8710)
3	4 → 5 by (1,1,2,2)	(37,44,54,68)	(4370,5385,6660,8490)
4	3 → 4 by (1,3,5)	(36,43,53,66)	(4420,5465,6760,8500)

From the above calculation, we can see that after the 4^{th} stage the total cost starts increasing. Hence, the total duration is (37, 44, 54, 68) and the corresponding total cost is (4370, 5385, 6660, 8490)

CONCLUSIONS

Fuzzy Critical Path length and Fuzzy Cost are the useful information for the decision makers in planning and controlling the complex projects. That is the decision maker can model their project and express the terms such as 'maybe', 'in between ' ,'nearly' and other linguistic variables for activity durations, whereas this specification do not exist in crisp models. In this paper, a new solution procedure for crashing network has been presented where the decision variables have been trapezoidal fuzzy numbers. A major advantage of this proposed approach is to eliminate negative and infeasible solutions which are being obtained by other methods. The validity of the proposed method is examined with a numerical example. The applicability of the above procedure is useful for various types of problems in which uncertain results exist.

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